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equation (1) $\sin \theta - m\theta = \sin \theta' - m\theta'$, and, if θ decreases, θ is found from (2) $\sin \theta + m\theta = \sin \theta' + m\theta'$. Suppose the rod is allowed to fall from the horizontal position $AB_0(\theta_0 = 0)$. It swings to the position $AB_1(\theta = \theta_1)$, then back to the position $AB_2(\theta = \theta_2)$, and, after a finite number of swings, comes to rest. Successive applications of equations (1) and (2) show that the angles θ_1, θ_2, etc., are given by the abscissas of the points P_1, P_2, etc., of Fig. 3. Each segment of the zigzag line $OP_1P_2P_3$ which is directed toward the right (OP_1,

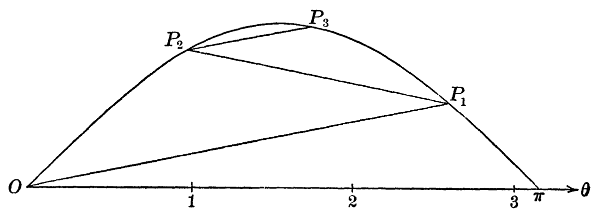

FIG. 3.

P_2P_3) has the slope m, and each segment which is directed toward the left (P_1P_2) has the slope $-m$. The figure is drawn for $i = 45°$, $\mu = 1/5$. By setting $\theta' = 0$ in equation (1) the value of θ_1 is found to be 2.595 radians. Hence the angle B_0AB_1 is equal approximately to 149°. By setting $\theta' = 2.595$ in equation (2) the value of θ_2 is found to be 0.996 radians. Hence the angle B_0AB_2 is equal approximately to 57°. By setting $\theta' = 0.996$ in equation (1) the value of θ_3 is found to be 1.733 radians. Hence the angle B_0AB_3 is equal approximately to 99°. A line drawn from P_3 with slope $-m$ will not intersect the arch of the sine curve. The force equations of the mechanical problem show that the rod will not move from this position.

Problem 3. The diameter of a bicycle wheel is 28 inches and the valve is at the lowest point of the wheel. The wheel is rolled forward until the valve is N inches ahead of its original position. Through what angle has the wheel turned? Assuming that the valve is 12 inches from the center of the wheel the equation to be solved is $N = 14\theta - 12 \sin \theta$.

Problem 4. Given a string wrapped around a circle. The locus of the end as it is unwound is the involute of the circle, $x = r \cos \theta + r\theta \sin \theta$, $y = r \sin \theta - r\theta \cos \theta$. Find the length unwound when x or y have given values.

Problem 5. The equation of a damped vibration has the form $x = ae^{-bt} \sin ct$. To find the time when the moving point is at a given distance D from the center, the equation would be put in the form $De^{bt} = a \sin ct$.

A DIFFERENTIATING MACHINE.

By ARMIN ELMENDORF,[1] University of Wisconsin.

A differentiating machine, as its name implies, is a device for drawing the differential or rate curve of any given curve, whether the latter be a curve plotted between two variables connected by an algebraic equation or an empirical curve obtained from experimental data. Its primary interest lies in its use for

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developing rate curves in various technological fields. The problem of rail friction, especially around curves, may be investigated by the use of a machine drawing the rate-of-change-of-velocity curve from velocity time data. From the rate or acceleration curve the friction force on the rails is readily computed by simple relations of mechanics. Many other instances could be cited in which rate curves are of great significance. By the use of the machine, load-deflection curves in impact on beams may be obtained, stresses in beams may be determined when the elastic curve is known, and from statistics such information as mortality and immigration rates become available.

As is well known, the slope of the tangent at any point on a curve is represented by the differential expression dy/dx when the curve is plotted in rectangular coördinates, and if it is desired to plot the rate dy/dx against the variable x it is necessary to determine the magnitude of the slopes by some mechanical means. This involves first the exact location of the tangent. The differentiating machine designed by the author uses for this purpose a small silver mirror which, when set vertically upon the curve, shows the image of the curve. When the image and the curve form a continuous line without a cusp at the junction of the curve and its image the reflector is exactly normal to the curve. Referring now to Fig. 1 in which it is assumed that the tangents have been exactly located,

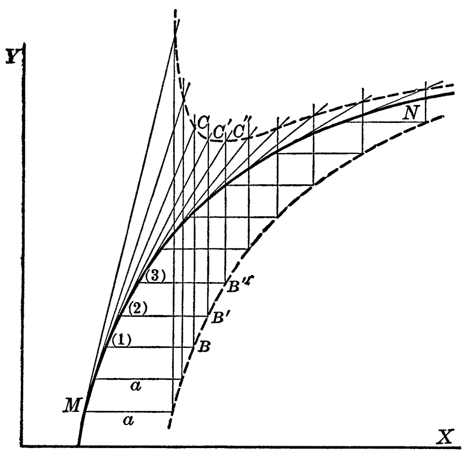

FIG. 1.

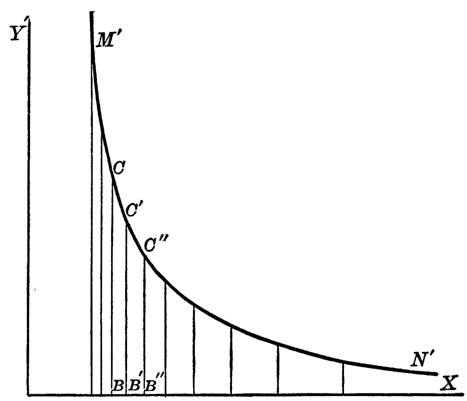

FIG. 2.

it is seen that, if a series of horizontal lines of constant length $a = 1$ be drawn from points along the given curve MN, the vertical distances intercepted by these horizontal lines and the tangents measure the slope at the various points. Thus BC represents the slope at the point (1), $B'C'$ that at point (2), etc. In Fig. 2 these distances have been plotted as ordinates giving the differential curve $M'N'$.

The next consideration is, then, the design of some device that will plot the variable distances between the two dashed lines of Fig. 1, as ordinates to a given base line.

The simple machine illustrated in Fig. 3 fulfills all the kinematic relations

embodied in the motions of the finished machine shown in Fig. 4. The mirror T is set so that the small scratch on the lower edge of the mirror which is vertically under the point O is upon the curve MN, thereby locating the tangent OB. The pin at B is free to slide in the tangent bar and also in the vertical arm EC. Link L is the base line of constant length. It is free to move in the horizontal slot of

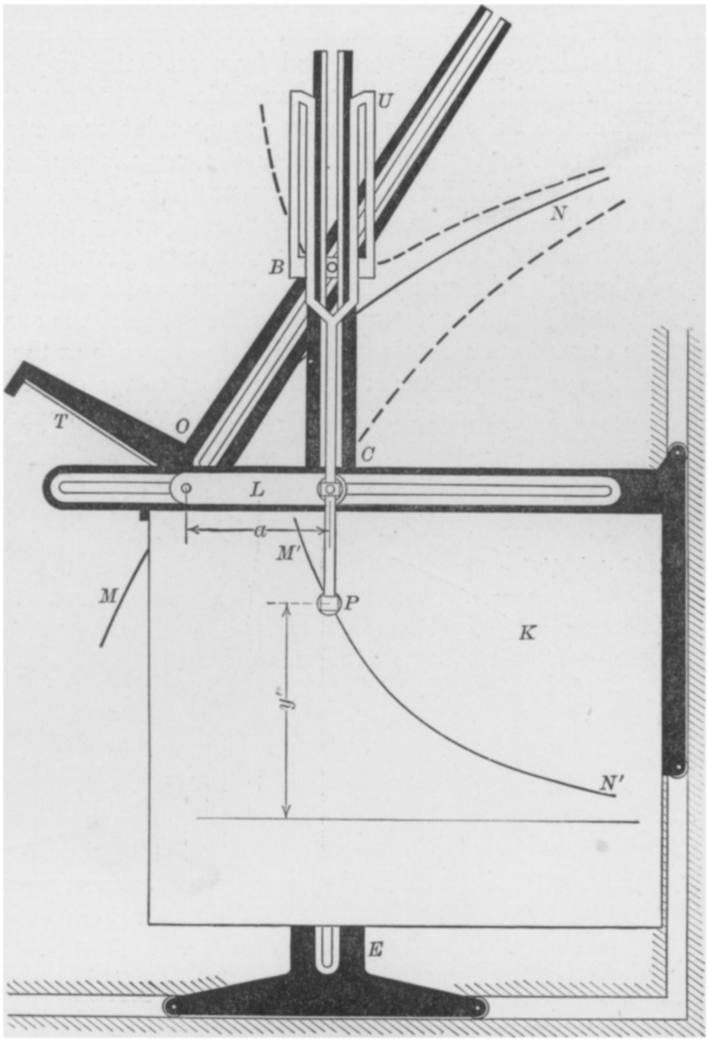

FIG. 3.

the carriage K, so that as the pivot O follows the curve MN, B traces the upper dashed line and C traces the lower line. Motion of the point B is transmitted through a yoke U, Fig. 3, to the tracing point P, enabling the pin B to pass under the point C as happens when the slope changes over from positive to neg-

ative values. When the pin B is under C, the tangent bar is horizontal, indicating a zero slope, and P is down at the base line. As the slope is increased the distance BC is increased and P is drawn up a distance equal to BC; in other words y', the distance drawn up, is in general the slope at some point on MN, or $M'N'$ on the carriage is the differential curve of MN. Fig. 4 clearly shows the

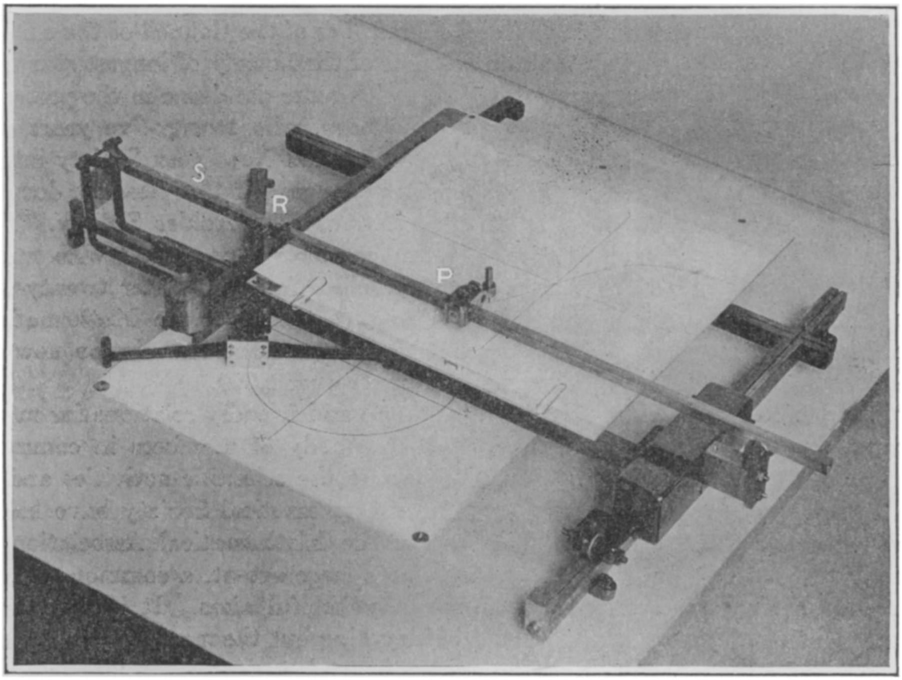

FIG. 4.

two grooves at right angles, one for the carriage and the other for the vertical arm. The mirror is set upon a circle. The tube S is graduated on its upper face so that the operator may obtain the numerical value of the slope at any point by noting the reading under the indicator at R.

In practice the best results are obtained by plotting a series of points on the differential curve and subsequently drawing a smooth curve through these points by hand. The machine has a slope capacity of about 4.5 but this may be changed by varying the length of the link a. For slopes of unity or less fairly smooth continuous curves may be drawn after some experience. The difficulty of obtaining a smooth continuous differential curve will probably remain as an imperfection in all differentiating machines, however perfect mechanically, when the location of the tangent is left to the operator. However, the "point-by-point" method, as a great many tests have verified, gives very accurate results, and only the time lost in filling in the curves by hand would be gained by the continuous process.